News and Macroprudential Policy

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February 2018

Abstract

Motivated by the deregulation of U.S. credit markets at the turn of the century, we analyze the cyclical properties of constrained optimal debt taxation in a quantitative model with systemic externalities. We focus on shocks to future income (news shocks), a salient feature of the U.S. economy during the late 1990s. In good times (positive news), it is optimal to allow for more borrowing in order to allow for consumption smoothing. When borrowing reaches a threshold, the economy enters a region where crises can occur. This pushes the Ramsey planner to tax borrowing. Thus, the constrained planner taxes borrowing in good times and when debt accumulation is high enough. Instead, in bad times, no taxation is necessary: agents anticipate that their income will be low and they save, escaping the possibility of a crisis. We contrast our findings to the case of standard, contemporaneous, shocks to income. Whereas under news shocks it is necessary to tax debt in good times, under contemporaneous shocks it is necessary to tax debt in bad times, when agents dig into their precautionary savings to smooth consumption. In a quantitative application to the U.S. economy from 1990 to 2015 we find that about half of the household leveraging can be judged as socially optimal from the perspective of a benchmark model.

Keywords: Macroprudential Policy; Financial Crises; Pecuniary Externality
JEL Classification: E32; E44; G18

[*]University of Oxford; [‡]Brandeis University; and [†]EIEF. This paper circulated previously with the title "The Optimal Tradeoff Between Consumption Smoothing and Macroprudential Regulation". We thank Javier Bianchi, Dan Cao, Giancarlo Corsetti, Andrea Ferrero, Jennifer La'O, Guido Lorenzoni, Hugo Hopenhayn, Kiminori Matsuyama, Guillermo Ordoñez, Edouard Schaal, César Sosa Padilla, and conference participants at SED for useful comments.

1 Introduction

During the late 1990s, policy makers in the U.S. took a number of important steps to deregulate financial markets, the most important step being the repeal of the Glass-Steagall Act by the Financial Services Modernization (Gramm-Leach-Bliley) Act in 1999. These policies have been highly controversial due to the subsequent build-up of leverage by American households and the ensuing financial crisis. Our goal in this paper is to make an attempt to understand the underlying economic conditions that motivated this push towards laissez-faire. We point out that a simple explanation to this wave of deregulation is based on the unusually favorable economic environment of the 1990s. Indeed, deregulation could actually have been the optimal response of policy makers to the optimistic beliefs about future income induced by this economic environment. In sum, there seems to be an important distinction in the assessment of the deregulation with and without the benefit of hindsight.

Motivated by this idea, in this paper we analyze a benchmark model that allows us to compute the optimal amount of financial regulation in the presence of advance information about shocks to future income (called news shocks in the literature, see Beaudry and Portier 2006, Jaimovich and Rebelo 2009, Schmitt-Grohe and Uribe 2012, among others). We analyze first a simple theoretical model that emphasizes a couple of interesting mechanisms in this context. We then generalize this into a full-blown quantitative model building on the influential work by Bianchi (2011). The model features a systemic externality that sets the stage for the analysis of constrained optimal (Ramsey) policy under rational expectations.

We first use a calibrated version of the model to quantify the optimal amount of borrowing that a constrained planner would allow for conditional on the characteristics of the U.S. economy in the mid-to-late 1990s. For this, we take an estimated series of news shocks for the U.S. from Blanchard, L'Huillier, and Lorenzoni (2013). Conditional on these shocks, the Ramsey planner optimally allows for more borrowing. The increase in borrowing is persistent and large. In particular, the predicted leveraging process takes about 5 years to be completed. Socially optimal leverage increases by 23%. The model accounts for 45% of the observed total household leveraging as the result of the optimal policy of a Ramsey planner subject to the news shocks of the mid-to-late 1990s.[1] The conclusion is that the news channel in a benchmark model predicts that about half of the observed leveraging was socially optimal, with the remaining being the result of agents failing to internalize the systemic externality of their borrowing choices, the effect of the unusual rise in house prices facilitating mortgage lending, and other channels not present in this benchmark model.

[1]In the U.S., household leverage increased by 47% (from a 62% of GDP in 1995 to 91% of GDP in 2006.)

We then explore the implicit taxes that implement the constrained planner's solution in the decentralized equilibrium. Here, we note that news shocks have drastic implications for macroprudential policy. To see this, consider first the cyclical implications of the benchmark by Bianchi (2011). There, the endowments follow a stochastic process subject to temporary shocks. In this case, by consumption smoothing, agents dig into their savings following a negative shock anticipating that the endowment will (mean-)revert in the future. Thus, agents borrow in bad times. This implies that the Ramsey planner needs to tax borrowing strongly in bad times, in order to make agents internalize the systemic risk implied by their borrowing decisions.

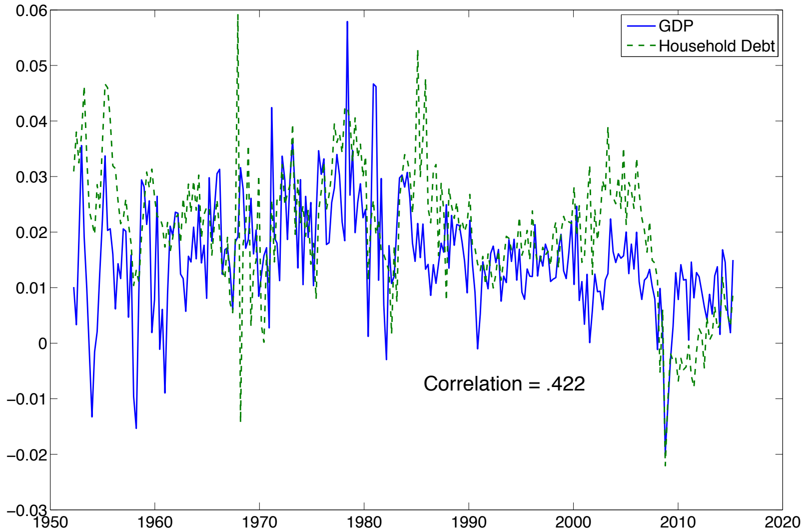
In contrast, consider now a Bianchi (2011) model with news shocks instead of temporary shocks. To stick as close as possible to the Bianchi (2011) model, we use the idea by Blanchard et al. (2013) of modeling news shocks as permanent and persistent shocks to the endowment, the idea being that a positive small shock grows over time, implying a large increase in income at infinity. The key insight is that now, by consumption smoothing, following a negative shock agents anticipate that their future income will be lower, causing them to save. Thus, in this case, the incentives of the agents and the incentives of the planner are aligned. Instead, by the same mechanism, agents borrow following a positive shock. This provides a reason for the Ramsey planner to tax. To sum up, we establish that under permanent shocks macroprudential policy is procyclical, whereas under temporary shocks macroprudential policy is countercyclical.

Our results regarding the taxes that implement the Ramsey planner's solution underline the importance of applying the right theory of debt in order to analyze macroprudential policy. Do agents borrow in good or in bad times? The news shocks view that agents borrow in good times is given support by the evidence depicted in Figure 1, which plots the quarterly growth rates of GDP and total household debt over the period 1952-2015. Consistent with findings in the literature (Beaudry and Portier 2006; Jaimovich and Rebelo 2009), positive news would have a procyclical effect on GDP, and would also push agents to borrow in anticipation of further GDP increases in future periods. The two series in the plot exhibit a clear positive relationship, with a correlation of 0.42.

Another important difference between temporary and permanent shocks regards the frequency of optimal debt taxation. In fact, the permanent shocks case features an unconditional probability of strictly positive taxation of 0.02, whereas this probability is 0.31 in the temporary shocks case. Thus, the news or (persistent) permanent shocks view of the household dynamics of debt assigns a different role to macroprudential policy in which it should be used much less often.

The paper proceeds as follows. Section 2 presents the simple model used for qual-

Figure 1: Quarterly Growth Rates, GDP and Total Household Debt, 1952-2015



Notes: GDP is from the BEA. Total Household Debt is from the Federal Reserve Board of Governors, Financial Accounts of the United States, Table D.3. Both series are seasonally adjusted and expressed in quarterly growth rates. Correlation coefficient=0.422.

itative purposes. Section 3 presents the more general model used for quantitative purposes. Section 4 concludes.

2 The Simple 3-period Model

In this Section, our goal is to present a very simple toy model to shed light on the quantitative exercise below.

We present a stylized framework to theoretically illustrate the effect of news for macroprudential regulation. We show two results. First, we show that a positive news shock pushes a constrained planner to allow for more borrowing. In other words, increased borrowing is constrained efficient, even though this generates more welfare losses in crises episodes. Second, we show that the implicit tax that decentralizes the constrained planner's solution is strictly positive only in the case of positive news. The reason is that, in bad times (negative news), saving is optimal given that future income is expected to be low. This implies that the incentives of the constrained planner and the agent in the decentralized economy are aligned, and thus there is no need for taxation. The opposite holds in the case of (strong enough) positive news.

The model is based on the models by Mendoza (2002), or Bianchi (2011), with a number of simplifications. The main objective here is to study the solution of a

Ramsey planner in the presence of news. In order to capture the effect of news, we introduce shifts in the distribution of future income. Because we are interested in the impact of these shifts on the planners' solution it does not matter if the news are actually realized or not, which is the focus on much of the news literature (Christiano, Ilut, Motto, and Rostagno 2008). Thus, in what follows, we assume that actual and perceived shifts in the distribution of future income are the same.

The model is a standard consumption-savings problem with a collateral constraint and pecuniary externalities. We consider an endowment economy with two goods: an aggregate consumption good c, and housing services h. There are three periods, indexed by $t = 0, 1, 2$. Period utility of a representative household is given by

$$u(c_t) + u(h_t) \tag{1}$$

where $u(c_t) = \log(c_t)$, c_t is consumption at period t, and h_t is the flow of housing services enjoyed at t. We assume that the representative household borrows from the rest of the world at a constant interest rate r. The household's budget constraint at period t is

$$c_t + p_t h_t + b_{t+1} = e_t^c + p_t e_t^h + (1 + r)(1 + \tau_t)b_t + T_t \tag{2}$$

where e_t^c is the endowment of the consumption good, e_t^h is the endowment of housing at t, p_t is the price of housing at t (the price of the consumption good being normalized to 1), where b_{t+1} is external debt taken on at period t, τ_t are taxes (which can be determined optimally by the implementation of the solution to the Ramsey problem below), and T_t is a lump-sum transfer. We assume that $b_0 = b_3 = 0$.

Borrowing in period 1 is constrained by

$$b_2 \geq -\kappa p_1 e_1^h$$

where $\kappa > 0$ is a parameter.

The endowments of the consumption good in periods 1 and 2 are stochastic: $e_1^c = e_2^c$ can take three values, High ($e_1^c = e^H$), Medium ($e_1^c = e^M$), and Low ($e_1^c = e^L$), $e^H > e^M > e^L$, each with respective probabilities π^H, π^M, π^L, with $\pi^L = 1 - \pi^H - \pi^M$. The endowment of the consumption good in period 0 is constant and normalized to 1: e_0^c. The endowment of housing is also assumed to be constant and normalized to 1: $e_t^h = 1$, $\forall t$.

Our aim is to solve the Ramsey planner's problem in this economy. The following preliminary result is useful for this purpose.

Lemma 1 (Price of Housing in the Competitive Equilibrium) *In the compet-*

itive equilibrium of this economy

$$p_1 = \frac{c_1}{e_1^h} \tag{3}$$

The proof of this simple result is in the Appendix (together with a definition of the competitive equilibrium which is standard.) The explicit form of the expression for p_1 is a consequence of log-log preferences.

The Ramsey planner's problem (associated with the household's problem above) is to choose consumption c_t, housing services h_t, and bond holdings b_{t+1}, $t = 0, 1, 2$ to maximize

$$\max_{c_t, h_t, b_{t+1}} \mathbb{E}\left[\sum_{t=0}^{2} \beta^t (\log(c_t) + \log(h_t))\right]$$

subject to

$$c_t = e_t^c + (1+r)b_t - b_{t+1} \tag{4}$$

and

$$b_2 \geq -\kappa c_1$$

where $\beta < 1$ is the discount factor, and (4) is a resource constraint resulting in the market equilibrium from the budget constraint (2). Notice also that we have substituted for the price using (3) in the collateral constraint. We denote the planner's solution by b_{t+1}^*, c_t^*, h_t^*. Throughout the paper, we use the terms Ramsey planner or constrained social planner indistinctively.

We call the solution to this problem a constrained efficient allocation. Bianchi (2011) has shown, in a similar economy, that this allocation can be decentralized by choosing the appropriate set of taxes τ_t and transfers T_t, $t = 0, 1, 2$. A similar result holds in this economy, and it is proven in Appendix B.

We say that a transition to a distribution of the endowments $e_1^c = e_2^c$ induces a FOSD shift in the endowment if the new distribution first-order stochastically dominates the old distribution. We denote the planner's solution with the Old distribution by b_{t+1}^{*O}, c_t^{*O}, h_t^{*O}, and the planner's solution with the New distribution by b_{t+1}^{*N}, c_t^{*N}, h_t^{*N}. Also, we denote by μ_1^P the Lagrange multiplier on the collateral constraint of the planner.

We now state our first result. A first order stochastic shift in the probabilities of the states (positive news shock) pushes the Ramsey planner to allow for more borrowing. The reason is a standard consumption smoothing motive. Notice however that this result is not obvious, because when the planner becomes optimistic, he allows for more borrowing even though this generates a larger welfare loss if the constraint ends up binding. In fact, the planner here faces the following tradeoff. On the one hand, allowing for more borrowing exploits welfare benefits coming from consumption smoothing. On the other hand, this increases the costs of crises. The proposition shows that, in this simple model, news in the form of a first order stochastic shift causes the first of these two considerations to dominate.

Proposition 1 (Effect of FOSD shift) *If $\kappa(2+r) > 1$, a FOSD shift in the distribution of $e_1^c = e_2^c$ pushes the planner to allow for more borrowing, i.e. $b_1^{*N} < b_1^{*O}$. As a consequence, the welfare decline is larger when the constraint bids, i.e.*

$$u'(c_1)\Big|_{b_1^{*N},\ \mu_1>0} > u'(c_1)\Big|_{b_1^{*O},\ \mu_1^P>0}$$

Our next results can be cast out more transparently in the absence of uncertainty. Thus, for simplicity, we remove uncertainty in what follows (i.e. the endowments e_0^c, e_1^c, e_2^c are drawn by nature at the beginning of period 0.)

Next we state our second result, which concerns the implicit taxes that decentralize the constrained planner's solution in the case of news, which is simply the value of e_2^c in the absence of uncertainty. The results states that if positive news are sufficiently favorable, the Ramsey planner finds it optimal to tax borrowing to mitigate the externality present when the constraint binds. Thus, the constrained planner pushes households to take into consideration the externality caused by their behavior. Importantly, this does not happen for bad news: in that case, households save and in this model they do so in such a way that the externality is absent. Thus their incentives are aligned with the Ramsey planner's. To sum up, optimal macroprudential policy taxes borrowing in good times. These insights are largely in line with what happens in our quantitative exercise below.

Proposition 2 (Implicit Taxes under News Shocks) *Suppose that for $e_0^c = e_1^c = e_2^c = 1$ the constraint does not bind. There is a cutoff $\overline{e_2^c}$ such that:*

- *for $e_2^c \geq \overline{e_2^c}$, $\tau_0 > 0$,*
- *for $e_2^c \leq 1$, $\tau_0 = 0$.*

We show in the following lemma that our previous finding regarding the implicit taxes in the news case is the opposite of what happens in the context considered by the literature so far of temporary (non-stationary) shocks to the endowment (Bianchi 2011). The lemma shows that in this case, a positive shock generates no implicit taxes. Instead, a negative shock pushes the constrained planner to tax borrowing. The reason is that the saving/borrowing decision here is the opposite. By consumption smoothing, a positive shock increases savings as the agent saves the temporary windfall and consumes it gradually over time. A negative shock pushes the agent to borrow to smooth out the bad shock. To sum up, in the temporary shocks case, the Ramsey planner taxes in bad times. Our quantitative model below will show similar results in a more general environment.

Lemma 2 (Implicit Taxes under Temporary Shocks) *Suppose that for* $e^c_0 = e^c_1 = e^c_2 = 1$ *the constraint does not bind. There are cutoffs* $\overline{e^c_0}$ *and* $\underline{e^c_0}$*,* $\overline{e^c_0} > \underline{e^c_0}$*, such that:*

- *for* $e^c_0 \geq \overline{e^c_0}$*,* $\tau_0 = 0$*,*
- *for* $e^c_2 \leq \underline{e^c_0}$*,* $\tau_0 > 0$*.*

3 Quantitative Exercises

In this section, we quantitatively study a more general model of macroprudential regulation in the presence of news. The model now has an infinite horizon. This increases the complexity of the problem significantly, and thus we use standard numerical methods for the analysis.

A natural way to introduce news in this type of model is to use persistent shocks to the growth of the endowments (as opposed to shocks to the level of the endowments), also named 'permanent' shocks or 'trend' shocks in the literature. Indeed, following Blanchard, L'Huillier, and Lorenzoni (2013), we will use the idea that a persistent permanent shock induces a fairly large change in the future level of the endowment. A particularly appealing feature of this approach is that it will immediately allow us to compare our findings to what is obtained in the case of shocks to the level, which is the standard assumption in the quantitative macroprudential literature up to date. Actually, as explained in detail in this section, growth shocks induce a dramatically different debt behavior than do level shocks, and thus have drastically contrasting implications for macroprudential policy.[2]

Besides allowing for a general characterization of optimal constrained policy in the presence of news, there is a second reason for doing this quantitative exercise. Below, we will be able to apply our quantitative model to the economic developments observed in the U.S. economy from 1990 onwards. There are four salient features of the U.S. economy since 1990 that our model will be able to speak to: first, the high growth seen roughly from 1995 to 2003 (Fernald 2015); second, the important deregulation of credit markets happening during the late 1990s; third, the sustained leveraging of U.S. households up to 2006; and fourth, the 2008 financial crisis. Our quantitative framework will help to conceptually organize these events. The model will be used to compute the constrained optimal amount of debt *conditional* on the observed growth

[2] Researchers have considered other way of introducing news into business cycle models, see Christiano, Ilut, Motto, and Rostagno (2008), Jaimovich and Rebelo (2009), or Schmitt-Grohe and Uribe (2012), among others.

from 1995 to 2003. Because the constrained optimal level of debt can be implemented in a decentralized equilibrium, this will have direct implications for the constrained optimal regulation of overborrowing during this episode.

This section is organized as follows. First, the model is presented. Second, the calibration of the model is discussed. Third, the model is simulated in order to explain the constrained optimal behavior of savings in the presence of growth shocks, and to characterize the taxes that decentralize this constrained optimal solution. Finally, the application to the U.S. is discussed.

3.1 Recursive Model

The recursive model generalizes the simple model outlined in the previous Section in several dimensions. We consider an infinitely lived representative household with period utility

$$(1-\alpha)\log(c_t)+\alpha\log(h_t)$$

where the parameter $\alpha \in (0,1)$. As explained below, because we want to specify a process for the endowments with permanent shocks and thus featuring a unit root, our utility specification for the consumption good c_t is restricted to the logarithm. This allows us to transform the model and achieve a bounded choice set.

The household is subject to the same budget constraint as above (2), and subject to a borrowing constraint every period given by:

$$b_{t+1} \geq -\kappa^h p_t e_t^h - \kappa^c e_t^c \tag{5}$$

where κ^h and κ^c are strictly positive parameters designating the proportion of the value of each endowment (of the consumption good or of housing) that can be pledged as collateral. Notice that, in contrast to the simple model of the previous section, here we follow Bianchi (2011) and allow the household to pledge both endowments as collateral. (This turns out to be important quantitatively.)

The endowment of the consumption good e_t^c follows the process

$$e_t^c = e_{t-1}^c \exp(\varepsilon_t) \tag{6}$$

where ε_t is therefore growth rate of the endowment between $t-1$ and t. This growth rate ε_t follows a Markov process, to be fully specified below. Notice that (6) implies that the logarithm of endowment $\log(e_t^c)$ has a unit root:

$$\log(e_t^c) = \log(e_{t-1}^c) + \varepsilon_t$$

The endowment of housing is proportional to that of consumption by a strictly positive factor γ: $e_t^h = \gamma e_t^c$.

As in the simple model of Section 2, we are interested in the problem of a Ramsey planner that is constrained by prices in the competitive equilibrium and by the borrowing constraints (5). A result similar to Lemma 1 establishes that, also in this version of the model, the equilibrium price p_t can be obtained in closed form as a function of consumption and the housing endowment

$$p_t = \frac{\alpha}{1-\alpha}\frac{c_t}{e_t^h} \tag{7}$$

Thus, the recursive formulation of the Ramsey planner's problem is

$$V_t(b_t, e_t^h) = \max_{c_t, h_t, b_{t+1}} (1-\alpha)\log(c_t) + \alpha\log(h_t) + \mathbb{E}\left[V_{t+1}(b_{t+1}, e_{t+1}^h)\right]$$

subject to

$$c_t = e_t^c + (1+r)b_t - b_{t+1}$$

and

$$b_{t+1} \geq -\kappa^h \frac{\alpha}{1-\alpha} c_t - \kappa^c e_t^c$$

where we have substituted for the price p_t by (7).

We solve this problem numerically by value function iteration. In order to do so we first need to transform the model to achieve bounded choice sets in the discretized model. The precise issue with using the non-transformed model is the following. As usual, we approximate the solution of the problem by defining grids for choice variables. Since the process for the (logarithm of the) endowment e_t^c has a unit root, the space of realizations of the endowment is unbounded, and choice variables are highly likely to attain values beyond the borders of the grid. Thus, the discretized solution provides a poor approximation of the actual solution of the non-transformed planner's problem. The transformation of the model below takes care of this problem.

3.2 Transformed Model and Calibration

Transformed Model. In order to transform the model, we define three variables $\tilde{c}_t$, $\tilde{h}_t$, and $\tilde{b}_{t+1}$ as follows:

$$\tilde{c}_t \equiv \frac{c_t}{e_t^c}$$

$$\tilde{h}_t \equiv \frac{h_t}{e_t^c}$$

$$\tilde{b}_{t+1} \equiv \frac{b_{t+1}}{e_t^c}$$

Period utility can be written in terms of the transformed variables as $(1-\alpha)\log(\tilde{c}_t e_t^c) + \alpha\log(\tilde{h}_t e_t^h)$, which is equal to

$$(1-\alpha)\log(\tilde{c}_t) + \alpha\log(\tilde{h}_t) + (1-\alpha)\log(e_t^c) + \alpha\log(e_t^h)$$

The last two terms of this expression do not depend on choice variables and therefore can be dropped from the maximization problem, obtaining period utility

$$(1-\alpha)\log(\tilde{c}_t) + \alpha\log(\tilde{h}_t) \tag{8}$$

The budget constraint, written in terms of the transformed variables, is

$$\tilde{c}_t e_t^c = e_t^c + (1+r)\tilde{b}_t e_{t-1}^c - \tilde{b}_{t+1} e_t^c$$

Diving both sides by e_t^c gives:

$$\tilde{c}_t = 1 + \frac{1+r}{\exp(\varepsilon_t)}\tilde{b}_t - \tilde{b}_{t+1} \tag{9}$$

Inspection of this equation reveals that a positive growth shock ε_t reduces the interest rate paid by the planner on the transformed debt $\tilde{b}_t$ and thereby relaxes the budget constraint.

The collateral constraint, written in terms of the transformed variables, is

$$\tilde{b}_{t+1} e_t^c \geq -\kappa^h \frac{\alpha}{1-\alpha}\tilde{c}_t e_t^c - \kappa^c e_t^c$$

and thus dividing both sides by the endowment e_t^c one obtains

$$\tilde{b}_{t+1} \geq -\kappa^h \frac{\alpha}{1-\alpha}\tilde{c}_t - \kappa^c \tag{10}$$

Notice, then, that (8), (9), and (10) define the transformed model in terms of variables $\tilde{c}_t$, $\tilde{h}_t$, and $\tilde{b}_{t+1}$. The endowment does not enter the model any longer, but only the current growth rate shock. In fact, it seems that with any other utility function it would not be possible to obtain utility maximization, in terms of $\tilde{c}_t$ independent of e_t^c. Lastly, notice also that the endowment of housing e_t^h does not enter the transformed problem. The reason is that the price is inversely proportional to this endowment, and thus it cancels out from the housing collateral in (10).

Calibration. Our primary goal in this calibration is to study the behavior of optimal constrained policy for a reasonable parametrization of the model. Thus, we take a normative standpoint, and do not intend to fit any particular *in vivo* data set. We aim to study the optimal dynamics of debt around growth shocks, and their implications for macroprudential policy. The set of parameter values to calibrate is given by $\{\beta, r, \kappa^h, \kappa^c, \alpha\}$, and the parameters of the Markov process for the growth shocks ε_t. We calibrate the model by using standard values used in the literature and suggested by empirical studies outside our exercise. In particular, in our benchmark calibration we will focus on the U.S. economy from 1990 to 2010 and take a plausible sequence of growth shocks (taken once again from the literature) to study what the model tells us regarding optimal policy in this particular case. However, as we will argue below, this is in principle just an application and the lessons of the model are more general.

The model is simulated at quarterly frequency, which facilitates the calibration of the Markov process for growth shocks by comparison to the business cycle literature. To ease the comparison to the quantitative macroprudential and debt dynamics literature (e.g. Bianchi 2011, Arellano 2008), we then present annualized figures from the output of the simulation.

The period length is a quarter. The annual interest rate is set at 4 percent per annum ($r = 0.0099$ or 0.99 percent per quarter), a standard value. We set the discount factor β to 0.96 per annum (0.9898 per quarter), a relatively standard value for business cycles models (as for instance the real business cycles model). As usual in models with incomplete markets, we check that $\beta(1 + r) < 1$ (which in our case is very close to 1) in order to ensure that agents hold debt in equilibrium.[3]

The credit coefficients entering (10), κ^h and κ^c, are set to the same values used by Bianchi (2011), $\kappa^h = \kappa^c = 0.32$. According to the Bureau of Labor Statistics (BLS)[4] the share of housing expenditures for American households in 1999 was estimated, on average, as 32.6%. This estimate is computed using the Consumer Expenditure Survey. This fraction ranges from 31.9% for renters to 34.6% for homeowners, and from 32.0% for non-single to 36.7% for single households. We set α to 0.33, implying in our model that the representative household spends one third of the total value of the endowments in housing.

Table 1 summarizes the calibration of the set of parameters $\{\beta, r, \kappa^h, \kappa^c, \alpha\}$.

In a paper targeted to estimate a news model using U.S. data, Blanchard et al. (2013) (henceforth BLL) estimate an AR(1) process for permanent (or growth) shocks using quarterly data for the U.S. economy. Our calibration of the Markov process for ε_t will be based on their results. We will specify the same process for growth in our model and study the behavior of the planner under the calibration of Table 1.

[3] See, for instance, Chamberlain and Wilson (2000) for a discussion of this point.

[4] "Housing expenditures". *Issues in Labor Statistics*, Summary 02-02, 2002. U.S. Department of Labor, Bureau of Labor Statistics.

Table 1: Calibration

Parameter	Value	Source
Interest rate	$r = 1.04^{1/4} - 1$	Standard value
Discount factor	$\beta = 0.96^{1/4}$	Standard value
Credit coefficient housing	$\kappa^h = 0.32$	Bianchi (2011)
Credit coefficient consumption good	$\kappa^c = 0.32$	Bianchi (2011)
Share of housing expenditures	$\alpha = 0.33$	BLS (2002)

Specifically, growth shocks follow an AR(1) process

$$\varepsilon_t = \rho\varepsilon_{t-1} + \eta_t \tag{11}$$

where ρ is a persistence parameter in $[0, 1)$, and ε_t is an i.i.d. growth shock drawn from a Normal distribution with mean zero and variance σ^2. Because of the unit root in the logarithm of the endowment e_t^c in (6), the growth shock ε_t is, once again, a "permanent shock", which is the name this shock receives in the business cycles literature[5].

BLL present two estimates of (11) for the U.S. economy, one based on a simple permanent income consumption model, and another based on a medium-scale DSGE.[6] Both estimations point to a highly persistent process (0.89 and 0.94). In this calibration we take the second value which corresponds to a the richer, medium-scale DSGE.[7] The corresponding value for the standard deviation of η_t is 0.07 percent[8].

3.3 Results: Inspecting the News Mechanism

We start by studying the behavior of constrained optimal borrowing in a basic scenario, representative of our application to the U.S. below. Suppose there is a one-standard-deviation positive permanent shock (news shock), and that the economy remains there for 40 periods. In this case, the economy features a large and persistent accumulation of debt. This is shown in Figure 2. The simulation starts with an initial condition for $\tilde{b}_t$ equal to its ergodic mean. The Figure shows is a total accumulation of debt per annual unit of the endowment of the consumption good of roughly 19 percent. This accumulation is sustained and takes about 5 years to be completed.

To understand these debt dynamics, Figure 3a plots the Ramsey planner's policy

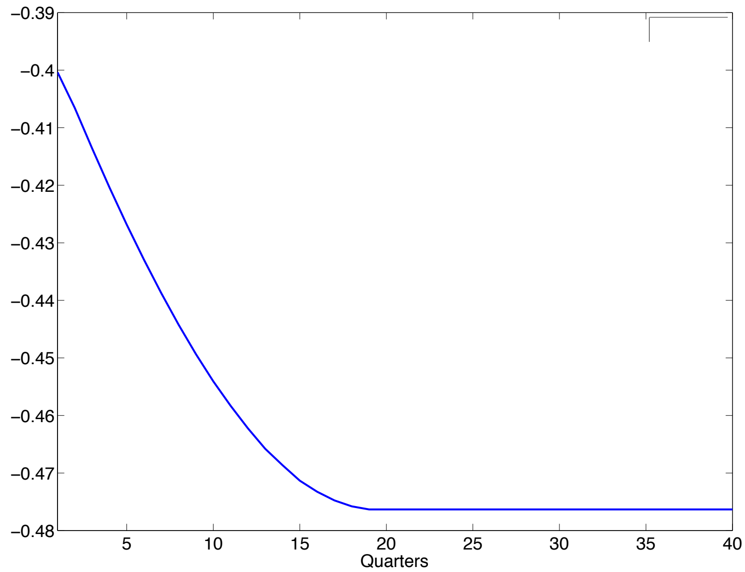
[5]See, besides BLL, Aguiar and Gopinath (2007), Boz, Daude, and Durdu (2011), or Cao and L'Huillier (2014) (among others).

[6]A bit of algebra shows that (6) and (11) are equivalent to process (2) of the permanent component in BLL (p. 3046).

[7]Using the first of these two values did not qualitatively change the results.

[8]In order to compute this value, take ρ and σ_u from Table 5 of BLL.

Figure 2: Conditional Dynamics of Debt after a Positive Permanent Shock

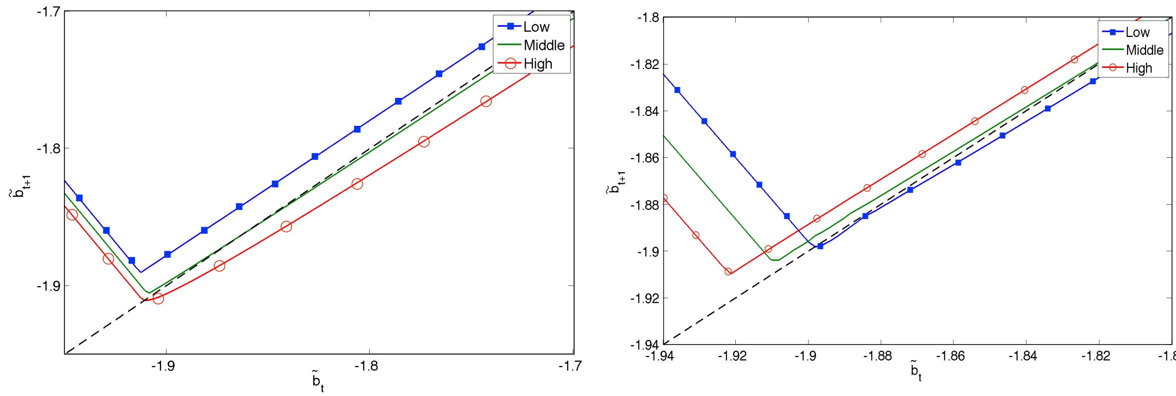


Notes: Simulation of constrained efficient annualized debt in response to a one-standard-deviation positive permanent shock lasting for 40 quarters. Debt corresponds to the path of $\tilde{b}_{t+1}$ *defined in Section 3.2.*

functions in three different states, which, for simplicity, we will call the High state, the Medium State, and the Low state. The first of these (High state) corresponds to the state of the economy when growth is a one-standard-deviation above zero. (This is the state used in the simulation above.) The second (Middle state) corresponds to the state of the economy when growth is at its mean: $\varepsilon_t = 0$. The third (Low state) corresponds to the state when growth is one-standard-deviation below zero. Each of these functions maps a current level of (transformed) asset holdings $\tilde{b}_t$ into next period's asset holdings $\tilde{b}_{t+1}$.

There are three main features of these policy functions to notice. First, and most importantly for our purposes, the higher the state, the lower the position of the policy function. That is, the top policy function corresponds to the Low state, the middle to the Medium state, and the bottom to the High state. This directly implies that the higher the state, the *more* the Ramsey planner borrows. Second, each of these functions is "V"-shaped, the upward-sloping region corresponding to unconstrained values of $\tilde{b}_{t+1}$, and the downward-sloping region corresponding to constrained $\tilde{b}_{t+1}$. Third, the middle policy function (Medium state) crosses the 45 degree line on the upward sloping region and for negative values of $\tilde{b}_{t+1}$ (the agent taking on debt into period $t+1$). The top policy function (Low state) does not cross the 45 degree line over the grid used for this simulation ($\tilde{b}_t \in [-2.1, 1]$), but may do so for very large values of $\tilde{b}_t$. The bottom policy function (High state) crosses the 45 degree line exactly at the kink.

Figure 3: Policy Functions of the Constrained Planner



(a) Permanent Shocks

(b) Temporary Shocks

Notes: Policy functions of the constrained planner for $\tilde{b}_{t+1}$ as a function of $\tilde{b}_t$ when the shock permanent (left) and temporary (right). The policy function labeled "Middle" corresponds to a value of ε_t equal to its mean. The policy functions labeled "Low" and "High" correspond to values of ε_t of one standard deviation below and above the mean, respectively.

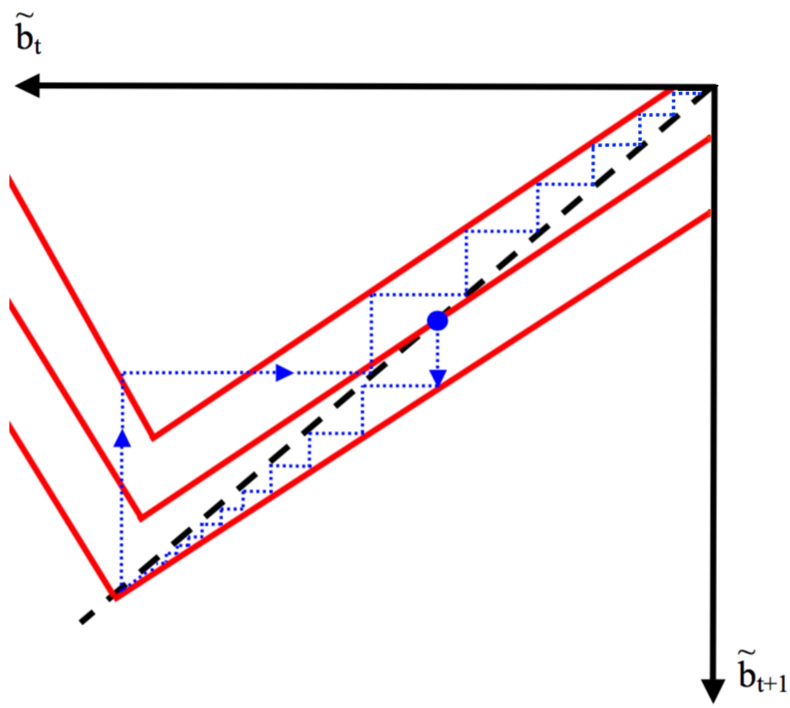
The shape of these policy functions contains rich information on the dynamics of debt in the model and on the implied implicit taxes that decentralize the Ramsey planner's solution. In order to see this, consider the schematic chart shown in Figure 4. In the example depicted in the chart, the dynamics begin at the fixed point of the middle policy function (denoted by the dot in the middle of the chart, where the Middle policy function crosses the 45 degree line). Then, the economy transits to the High state, and the planner begins accumulating debt. The reason is a news effect. This effect can be understood by recognizing that, in the High state, the logarithm of endowment is a supermartingale so long as the state is persistent:

$$E\left[\log(e^c_{t+1}) | \log(e^c_t), e^c_t = e^H\right] > \log(e^c_t)$$

Thus, the future endowment is expected to be high, and consumption smoothing pushes the planner to borrow.

In the example, the economy stays in the High state for a while, and there is convergence of debt accumulation to the fixed point in the High state. At some point the economy transits to the Low state, and the economy enters a crisis, which is defined (following the literature) as an episode in which the collateral constraint binds. Net

Figure 4: Schematic Dynamics of Debt



capital flows out ($-((1+r)\tilde{b}_t/\exp(\varepsilon_t) - \tilde{b}_{t+1}) > 0$) and the crisis immediately implies deleveraging. However, so long as the economy stays in the Low state for some time, the news effect implies that deleveraging will be sustained. The planner gradually repays his debt and can even accumulate assets ($\tilde{b}_{t+1} \geq 0$), unless the economy transits again to a higher state.

The constrained optimal dynamics of debt described previously have sharp implications for the taxes that implement the constrained planner's solution. First, because in the Low state the policy function is above the 45 degrees line, saving is optimal. Intuitively, this suggests that the incentives for the household in the competitive equilibrium and for the planner are aligned, and there is little or no need for financial regulation. In fact, given that the policy function is far above the 45 degree line (Figure 3a), one can guess that the agent will save enough to "escape" the possibility of a crisis. It turns out that this intuition is correct. This can be seen from the expression for the optimal tax which is[9]

$$\tau_t = \frac{E_t\left(\frac{\mu^P_{t+1}\kappa_h\frac{\alpha}{1-\alpha}}{exp(\epsilon_{t+1})}\right)}{E_t\left(\frac{1}{exp(\epsilon_{t+1})\tilde{c}_{t+1}}\right)}$$

This expression shows that if the probability of a crisis in the next period is zero (and thus $E[\mu^P_{t+1}] = 0$), then the optimal tax is zero. In the Low state, this is indeed the case (and therefore the tax is zero.) Second, in the High state and when the amount of debt entering the period $\tilde{b}_t$ is rather small, there are incentives to borrow and therefore there is in principle a need for regulation. However, a key insight from this quantitative analysis is that, because the kinks of the policy functions line up one on top of the other, the set of debt levels $\tilde{b}_t$ that can trigger a crisis is rather small. This interval, or 'tax region', is shown by the shaded portion of Figure 5a. Thus, even though there are strong incentives to borrow in the High state, the region where financial crises can occur is small, and therefore there is no need for government intervention unless the stock of debt is very high. Accordingly, the unconditional probability of taxation in the permanent shocks case is only of 5.7%.

To emphasize the novelty of these results, we next contrast our findings to the case of shocks to the level of the endowment, which is the standard case analyzed by Bianchi (2011), among others. To this end, we consider a variation of the model above with the process

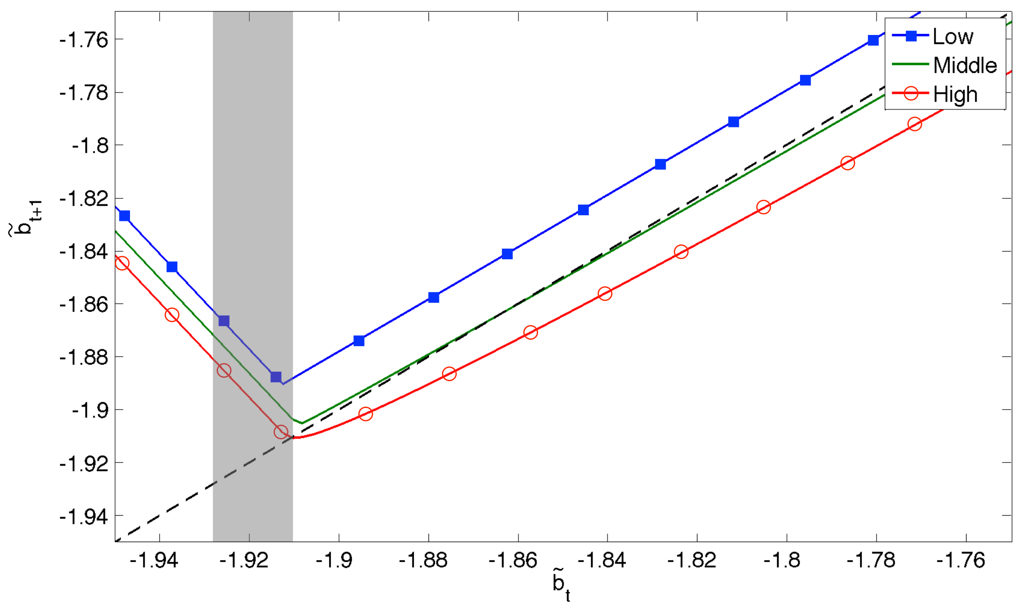
$$e^c_t = \exp(\varepsilon_t)$$

instead of (6). Notice that now the endowment no longer has a unit root (η_t in stochastic process (11) is thus a temporary shock.) Figure 3b shows the constrained planner's policy functions in this case, using the corresponding definition of the High, Middle, and Low states (the High state corresponding to one-standard-deviation in the log-level of the endowment above zero, the Middle state corresponding to zero, and the Low state corresponding to one-standard-deviation below zero.) A striking feature of these policy functions is their position, which is the opposite of what we obtained in the permanent shocks case. That is, in the temporary shocks case, the policy function in the High state is positioned at the top. In the Low state, the policy function is positioned at the bottom. Moreover, the kinks of the policy functions are no longer aligned vertically, which as we will explain, has direct implications for the

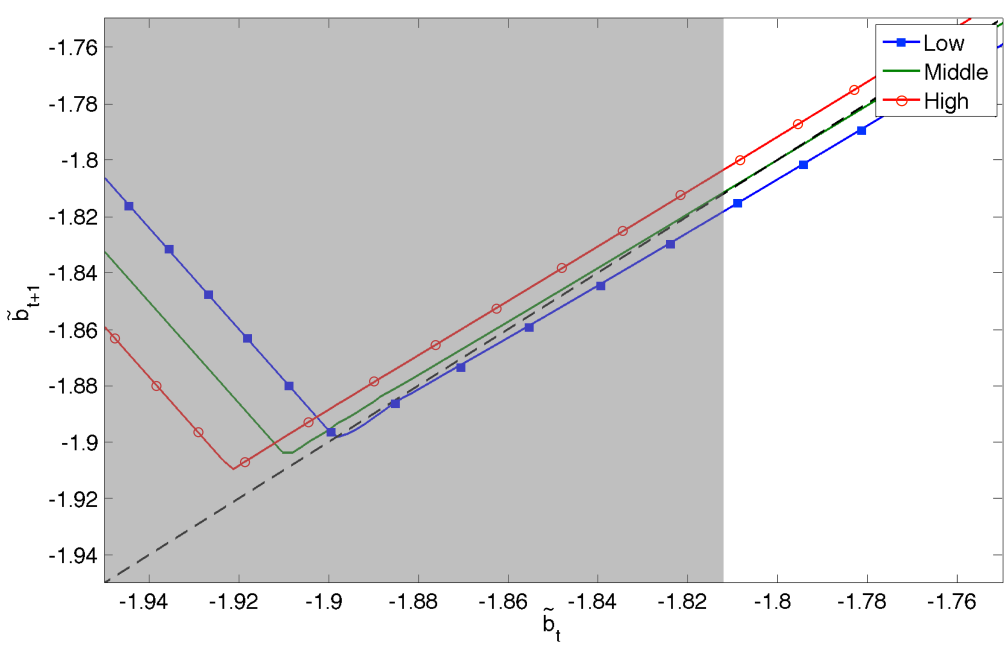
[9]See Appendix B for the derivation of this expression. As in Bianchi (2011), the derivation of a tax which equalizes the constrained planner's and the agents' first order conditions in the CE results in a more involved tax for tax levels $\tilde{b}_t$ at which the constraint binds for the planner, but we verify numerically that this simpler tax (which is obtained in the non-binding case) also implements the planner's solution. The intuitive reason is that in this case it is the constraint (10) that determines $\tilde{b}_{t+1}$.

Figure 5: Tax Regions and Policy Functions of the Constrained Planner

(a) Permanent Shocks



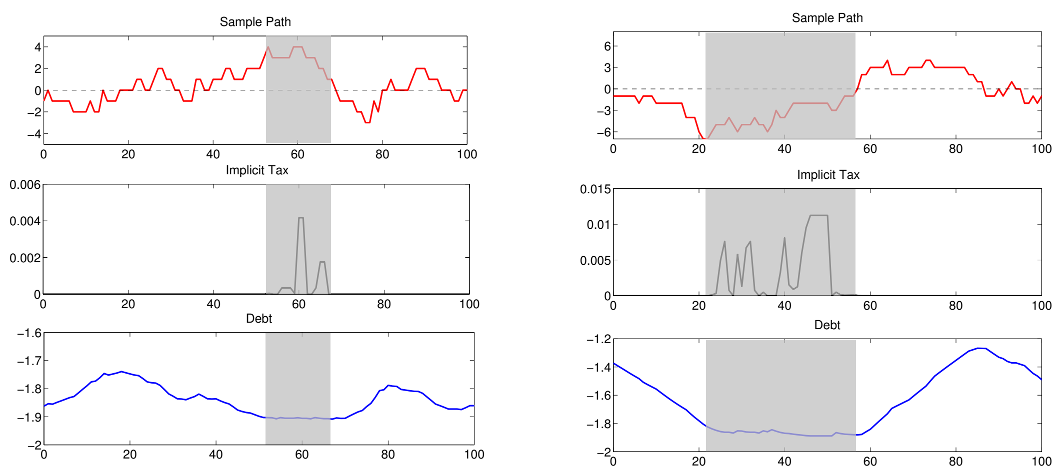
(b) Temporary Shocks



amount of time the constrained planner taxes in order to implement the constrained efficient outcome.

Given that in Figure 3b (temporary shocks) the policy function in the High state is above the 45 degrees line, the optimal action is to save. This is the exact opposite of what was obtained in the permanent shocks case. Under temporary shocks, precautionary savings pushes the constrained planner to save in good times. The opposite holds in bad times.

Figure 6: Sample Paths under Permanent and Temporary Shocks



(a) Permanent Shocks

(b) Temporary Shocks

Notes: The top panels show 25-year sample paths for shocks in both the permanent (a) and temporary (b) models, with the dashed line indicating the steady state value for both shocks. The center panels show the implied taxes necessary to implement the planner's choice of debt into the competitive equilibrium. The bottom panels show the paths of constrained efficient debt in response to the shocks. Negative values correspond to more debt. Shaded regions denote periods in which taxes are positive.

We illustrate the contrasting borrowing/saving behavior under permanent/ temporary shocks in Figure 4, which depicts sample paths, debt dynamics, and implicit taxes in both cases. Because in the temporary (permanent) shocks case the incentive to borrow is strongest in bad (good) times, the Ramsey planner taxes the household in the Low (High) state. Thus, there is a negative (positive) correlation of economic growth and optimal macroprudential regulation, i.e. optimal macroprudential regulation is countercyclical (procyclical).

Finally, notice that the tax region under temporary shocks is wider than in the case of permanent shocks. This suggests that the unconditional probability of the tax being positive is higher than under permanent shocks. This is indeed the case, with the economy featuring strictly positive taxes 28.3% of the time.[10]

[10] At this point, the reader may wonder if the permanent shocks case also differs strongly from the tem-

To sum up, we have obtained three main results. First, unless the stock of debt is so high that a crisis can occur, the constrained planner strongly increases borrowing in higher states of the world. The reason is an income effect which pushes the planner to allow for the benefits of consumption smoothing with expectations about future income are rosy. The Ramsey planner decides to do this even though he approaches the region where crises can occur. When crises can occur, the constrained optimal debt accumulation is small or zero. Second, under permanent shocks, the constrained planner taxes in good times, because then the incentives to borrow are strong. The opposite happens under temporary shocks: the constrained planner taxes in bad times, because a precautionary motive provides incentives to borrow. Third, the probability of taxation under permanent shocks is quite small. The reason is that crises occur for very high levels of debt accumulation.

3.4 Application to the U.S. Economy, 1990–2015

Our news mechanism implies that it is constrained optimal to allow for more borrowing in a state of positive news. We have also shown that persistent growth shocks can be thought of as implicitly having a positive news element. Thus, we can use our model to ask the following question: What is the optimal U.S. household leveraging during the 1990s, a period of remarkable and persistent GDP growth? Having answered this question, is the observed U.S. household leveraging above or below this figure?[11]

We proceed in two steps. First, we simulate the constrained optimal behavior of debt obtained by our model conditional on the growth shocks estimated in the literature (Blanchard, L'Huillier, and Lorenzoni 2013). Second, we compute the associated implicit taxed that implement the planner's solution as a competitive equilibrium. This computation will potentially allow us to interpret the planner's solution in terms of credit market regulation (or deregulation).

Figure 7 plots the (annualized) growth rate of the endowment e_t^c at each point in time implied by BLL.[12] This Figure shows that growth peaked in 1998, and then was persistent, but gradually declined following (11).

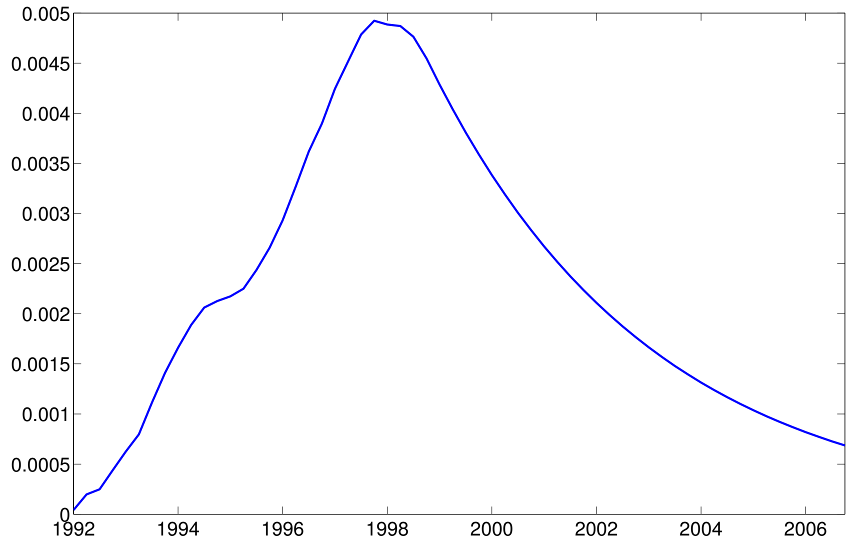
Figure 8 presents the associated conditional simulation of socially optimal debt

porary shocks case in terms of ergodic distributions of debt. It does not. Even though the annual ergodic mean debt per unit of endowment of the consumption good b_t/e_t^c is a bit lower in the case of temporary shocks (-0.36 instead of -0.39), the shape of both distributions is similar.

[11]See Fernald (2015) who documents the evolution of productivity growth in the 1990s, up to the financial crisis. See Cao and L'Huillier (2014) and Hoffmann, Krause, and Laubach (2011) for business cycle models on the same topic.

[12]Specifically, we proceed as follows. We re-estimate the model in Section 3 of BLL, and then use a Kalman smoother to estimate the permanent shocks ε_t. We record the quarter in which a positive shock first hit the U.S. economy during the 1990-2005 period (1991:Q4), and simulate the growth rates implied by this and subsequent shocks during the next 7 years, using (11).

Figure 7: Growth Rates Implied by the Shocks Computed by Blanchard et al. (2013)



dynamics. There is a sustained and fairly large accumulation of debt from roughly -0.40 to -0.48 that takes about 22 model periods (5.50 years). Thus, the model predicts that a substantial and persistent leveraging was optimal due to the sustained period of economic growth.

In the data[13], in 1992 the U.S. featured a total household debt-to-GDP ratio of 0.61, and the ratio increased to a maximum level of 0.92 in 2006. Thus, in the data households leveraged by more than what was optimal according to our calibrated model. This can be due to agents' not internalizing the externality of their borrowing decisions for the aggregate economy, or due to other imperfections not present in the model as the role of banks' incentives in the mortgage market.[14]

Figure 9 plots the implicit tax that decentralizes the planner's solution given the growth rates plotted in Figure 7. The figure shows that the planner allows households to borrow as much as they desire at the initial phase of leveraging (1992-1997), but strongly taxes borrowing when the economy enters the region where crisis can occur. This happens roughly 6 years after the start of leveraging, in 1998.

[13]Source: Authors' computations from series CMDEBT from the flow of funds. Sources: Federal Reserve Board of Governors, Financial Accounts of the United States and Bureau of Economic Analysis, respectively.

[14]The calibrated model is able to account for 45% of this debt increase in the data as a result of the optimal decision of a Ramsey planner (0.45=(0.48/0.39-1)/(0.91/0.62-1)). Of course, we did not intend to fully account for the total increase in debt, as we did not calibrate the model to match moments in the data. In this normative approach, we have used values that seem a priori reasonable based on previous literature (shown in Table 1 and from Blanchard et al. 2013). Also, part of the increase in debt in the data was certainly driven by elements outside our model, for instance the unusually high level of house prices (named bubble by some), or the facilitation of credit access through financial innovation.

Figure 8: Conditional Dynamics of Debt using the Shocks Computed by Blanchard et al. (2013)

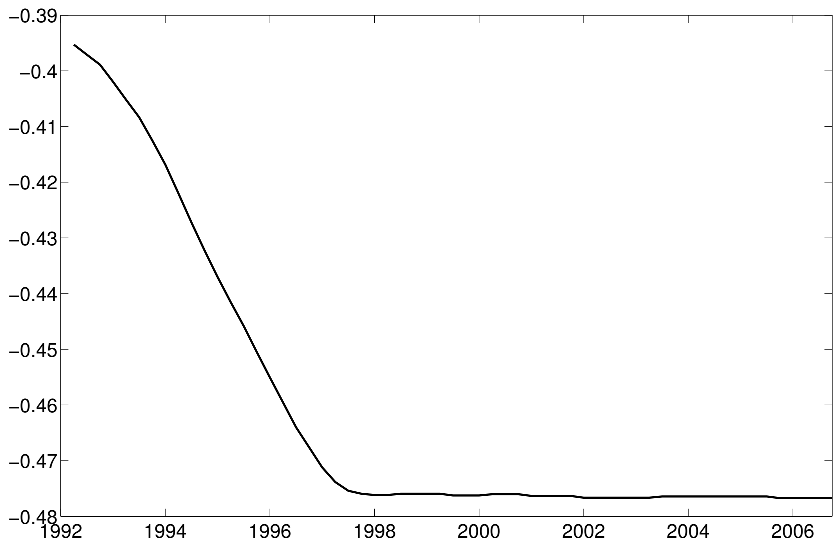
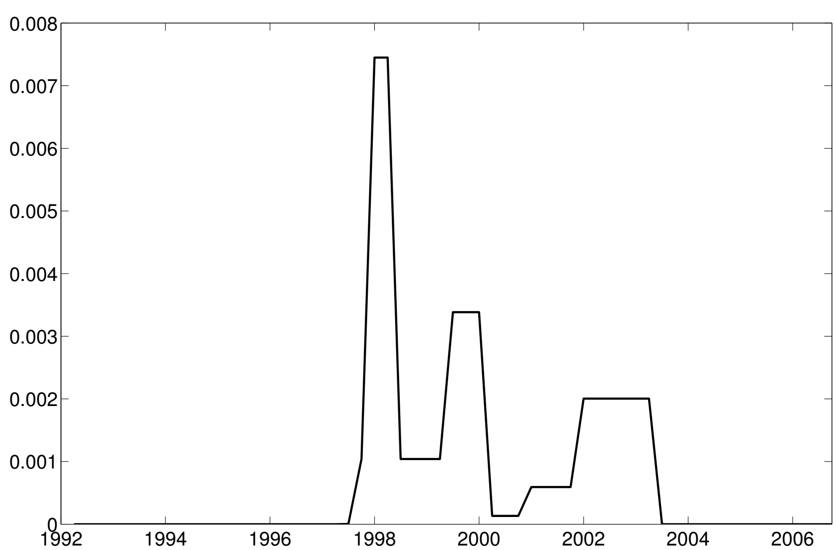


Figure 9: Taxes using the Shocks Computed by Blanchard et al. (2013)



Another dimension in which the simulated debt dynamics implied by our model do not match the actual dynamics of the U.S. economy is that the debt accumulation implied by the model happens much earlier (as shown in Figure 8). In the data, households' leverage being to increase only around the turn of the century. This could be the result of informational or financial frictions delaying the leveraging of U.S. households that our model does not account for (see Cao and L'Huillier 2014 for an in depth analysis of this point.)

4 Conclusions

In this paper we have analyzed the problem of macroprudential regulation in the presence of news about future income, or persistent permanent (growth) shocks. Positive news leads to optimally allowing for more borrowing. However, when the cumulated amount of borrowing is high enough, taxation of more borrowing is optimal in order to make agents internalize the systemic externality of their decisions. Moreover, taxation of borrowing is procyclical because there is little or no need of regulation in the case of negative news shocks. This is in contrast to the case usually analyzed in this literature so far of contemporaneous (temporary) shocks to income, where optimal taxation of borrowing is countercyclical. In an application to the U.S., we have taken a normative perspective and used a benchmark model to quantify the socially optimal amount of borrowing given the economy environment of the 1990s. We have found it was about half of what was actually observed.

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A Proofs

A.1 Definition of Equilibrium

Definition 1 *A decentralized equilibrium in the 3-period model is defined by a pricing function* $\{p_t\}_{t=0,1,2}$ *and borrowing choices* $\{b_{t+1}\}_{t=0,1}$ *such that*

1. *Borrowing choices* $\{b_{t+1}\}_{t=0,1}$ *solve*

$$\max_{\{b_{t+1}\}_{t=0,1}} \sum_{t=0}^{2} \beta^t (\log(c_t) + \log(h_t)$$

$$s.t. \quad c_t + p_t h_t + b_{t+1} = e_t^c + p_t e_t^h + (1+r)(1+\tau_t) b_t + T_t$$

$$b_2 \geq -\kappa p_1 e_1^h$$

   given $\{p_t\}_{t=0,1,2}$*, where* $b_0 = 0$*.*

2. *Markets clear: for* $t = 0, 1, 2$*,*

$$c_t = e_t^c + (1+r) b_t - b_{t+1}$$

$$h_t = e_t^h$$

A.2 Proof of Lemma 1

The household maximization problem is:

$$\max_{\{b_{t+1}, c_t, h_t\}} \sum_{t=0}^{2} \beta^t (\log(c_t) + \log(h_t)) \tag{12}$$

$$c_t + p_t h_t + b_{t+1} = e_t^c + p_t e_t^h + (1+r)(1+\tau_t) b_t + T_t$$

$$b_2 \geq -\kappa p_1 e_1^h$$

Denoting the Lagrange multipliers on the budget constraints λ_t and the Lagrange multiplier on the collateral constraint μ_1, the first order conditions of the household's problem are given by:

$$\frac{1}{c_t} = \lambda_t$$

$$\frac{1}{h_t} = p_t \lambda_t$$

$$\lambda_1 = \mu_1 + \beta E_1 \left[\lambda_2 (1+r)(1+\tau_2) \right]$$

Combining the first two equations for $t = 1$ and imposing the housing market clearing condition $h_1 = e_1^h$ gives the result.

A.3 Proof of Proposition 1

Suppose $e_1 = e_2 = e$ is a random variable from the perspective of $t = 0$. Let the "old" distribution be given by $\Pi = [\pi_L, \pi_M, \pi_H]$ and the "new" distribution be given by $\Pi' = [\pi'_L, \pi'_M, \pi'_H]$, where Π' FOSD Π. Denote the optimal choice of borrowing at $t = 1$ under the old distribution b_1^{*O} and the optimal choice of borrowing at $t = 1$ under the new distribution b_1^{*N}.

The planner's euler equation may be written as

$$\frac{1}{e - b_1^{*O}} = E\left(\frac{1}{e + (1+r)b_1^{*O} - b_2} + \kappa\mu_1\right) \tag{13}$$

where the expectation is taken over the distribution Π, with the multiplier on the borrowing constraint given by

$$\mu_1 = \frac{1}{1-\kappa}\left(\frac{1}{e + (1+r)b_1^{*O} - b_2} - \frac{1}{e + (1+r)b_2}\right)$$

There are three cases to consider.

1. If the constraint does not bind for any realization of e,

$$b_2 = \frac{(1+r)b_1}{2+r}.$$

   The right hand side of (13) is strictly decreasing in e. It follows that the planner borrows more under the new distribution relative to the old.

2. If the constraint binds for all realizations of e,

$$b_2 = -\kappa(e + (1+r)b_1)$$

   and the expression for the lagrange multiplier is:

$$\mu_1 = \frac{1}{e + (1+r)b_1} - \frac{\kappa}{(1-\kappa)e - \kappa(1+r)(e + (1+r)b_1)}$$

   or equivalently:

$$\mu_1 = \frac{1}{e + (1+r)b_1} - \frac{\kappa}{(1 - \kappa(2+r))e - \kappa(1+r)^2 b_1}.$$

   μ_1 is decreasing in e if

$$\frac{-1}{(e + (1+r)b_1)^2} + \frac{\kappa(1 - \kappa(2+r))}{(1 - \kappa(2+r))e - \kappa(1+r)^2 b_1)^2} < 0$$

A sufficient condition for the above inequality to hold is $\kappa(2+r) > 1$.

Note that μ_1 is decreasing in b_1 when $\mu_1 > 0$. It follows that if the constraint always binds and $\kappa(2+r) > 1$, the planner will borrow more under the new distribution.

3. By continuity, if the constraint is binding for some values of e and not binding for others, again a FOSD shift in the distribution of e induces more borrowing by the planner: $b_1^{*N} < b_1^{*O}$.

Turning to ex-post welfare at $t = 1$. If the constraint binds, $c_1 = \frac{e+(1+r)b_1}{1-\kappa}$ is increasing in b_1. Thus $c_1(b_1^{*N}) < c_1(b_1^{*O})$ for every realization of e for which $\mu_1 > 0$. Clearly, this implies

$$u'(c_1)|_{b_1^{*N},\mu_1>0} > u'(c_1)|_{b_1^{*O},\mu_1>0}$$

A.4 Proof of Proposition 2

Fix $e_c^0 = e_c^1 = 1$ and define a constant ε such that $e_2 = 1+\varepsilon$. At $\varepsilon = 0$, $b_1 = b_2 = 0$ with $\mu_1^P = 0$ and $\tau_0 = 0$.

Consider an arbitrarily small value of $\varepsilon > 0$ such that the borrowing constraint does not bind. The planner's first order conditions imply

$$b_2^* = \frac{(1+r)b_1^* - \varepsilon}{2+r},$$

$$b_1^*(\varepsilon) = \frac{-\varepsilon}{r^2+3r+3}.$$

Using the borrowing constraint, we derive the condition such that the constraint binds as

$$b_1 < \frac{(1-\kappa)\varepsilon - \kappa(2+r)}{(1+r)(1+\kappa(1+r))}.$$

Define $\overline{\varepsilon}$ as the unique value of ε such that

$$b_1^*(\overline{\varepsilon}) = \frac{(1-\kappa)\overline{\varepsilon} - \kappa(2+r)}{(1+r)(1+\kappa(1+r))}. \tag{14}$$

The tax that implements the constrained efficient borrowing choice in the decentralized equilibrium at $t = 0$ can by written

$$\tau_1 = \frac{\kappa\mu_1^P}{\frac{1}{c_1}}$$

Define $\overline{e_2^c} = 1 + \overline{\varepsilon} + \delta$ for an arbitrarily small $\delta > 0$. Suppose $\hat{\varepsilon} \geq \overline{\varepsilon} + \delta$. The planner's euler equation is given by

$$\frac{1}{e-b_1} = \frac{1}{1-\kappa}\left(\frac{1}{1+(1+r)b_1-b_2} - \frac{\kappa}{1+\hat{\varepsilon}-(1+r)b_2}\right). \tag{15}$$

If the constraint binds, (15) is equivalent to

$$\frac{1}{e-b_1} = \frac{1}{1+(1+r)b_1} - \frac{\kappa}{(1-\kappa)(1+\hat{\varepsilon})-\kappa(1+r)(1+(1+r)b_1)}. \tag{16}$$

Note that (16) implies that as $\hat{\varepsilon}$ increases, b_1 must also increase. It remains to show that it is optimal for the planner to choose b_1 such that the constraint binds.

For a contradiction, suppose that $\mu_1^P = 0$ at $e_2 = e + \hat{\varepsilon}$. Denote the optimal choice of borrowing at $t = 0$ as $\hat{b}_1$. If the constraint does not bind, then $\hat{b}_1 > b_1^*(\hat{\varepsilon})$. If the constraint does not bind, the planner is able to perfectly smooth consumption across all periods, implying a choice of $\hat{b}_1 = b_1^*(\hat{\varepsilon})$, which is a contradiction. It follows that for $e_2^c > \overline{e_2^c}$, $\tau_0 > 0$.

Now consider an arbitrary value of $\varepsilon < 0$ with $e_2 = 1 + \varepsilon$. The optimal choices of borrowing in periods 0 and 1 are given by:

$$b_1^*(\varepsilon) = \frac{\varepsilon}{r^2+3r+3}$$

$$b_2^* = \frac{(1+r)b_1^*+\varepsilon}{2+r}$$

Therefore, borrowing decreases in both periods in response to a decrease in e_2. Since μ_1^P is weakly decreasing in b_1 and e_2, it must be the case that the constraint will not bind for $e_2 = 1 - \varepsilon$ if it does not bind for $e_2 = 1$. Thus for $e_2 \leq 1$, $\tau_0 = 0$.

A.5 Proof of Lemma 2

Fix $e_1^c = e_2^c = 1$. Let $e_0^c = 1 + \varepsilon$ for an arbitrary constant ε. The euler equation for the constrained planner is given by

$$\frac{1}{1+\varepsilon-b_1} = \frac{1}{1+(1+r)b_1-b_2} + \kappa\mu_1^P \tag{17}$$

where b_1 is chosen such that (17) holds and μ_1^P denotes the value of the lagrange multiplier given $e_1^c = e_2^c = 1$.

It is straightforward to show that an increase in the value of ε implies that the optimal choice of debt at $t = 0$ must decrease in order for the euler equation to hold.

For $e_0^c = 1$, $b_1 = b_2 = 0$ and $\mu_1^P = 0$. For any value of e_0^c, the unconstrained choice of b_1 satisfies the following condition:

$$b_1(e_0) = \frac{(e_0^c - 1)(2 + r)}{r^2 + 3r + 3}$$

which is increasing in e_0^c.

It follows that it is optimal to save for $e_0^c > 1$. Since the values of the endowment at $t = 1, 2$ do not change, $\exists\ \hat{e_0^c} < 1$ such that

$$b_1^*(\hat{e_0^c}) = \frac{-\kappa(2 + r)}{(1 + r)(1 + \kappa(1 + r))}. \tag{18}$$

where $b_1^*(\hat{e_0^c})$ denotes the level of b_1 below which the constraint binds. Define $\underline{e_0^c} = \hat{e_0^c} - \delta$ for an arbitrarily small $\delta > 0$ and $\overline{e_0^c} = \hat{e_0^c}$. Then $\tau_1 > 0$ for $e_0^c \leq \underline{e_0^c}$ and $\tau_1 = 0$ for $e_0^c \geq \overline{e_0^c}$.

B Decentralization of the Planner's Solution

Result: The constrained efficient allocation chosen by the planner can be implemented in the decentralized equilibrium with a state contingent tax on debt, rebated to households as a lump sum transfer

Proof.

(i) Following Bianchi (2011, Proposition 2), we first consider the constrained efficient equilibrium:

The constrained efficient allocations are characterized by $\{c_t, h_t, b_1, b_2, p_t, \mu_1^P\}_{t=0,1,2}$ given initial debt $b_0 = 0$ such that the planner's euler equations are satisfied, $\mu_1^P \geq 0$, and the resource constraints and market clearing price hold.

The planner's euler equation between $t = 0$ and $t = 1$ is given by

$$\frac{1}{e_0^c - b_1} = \beta(1+r)E\left[\frac{1}{e_1^c - b_2 + (1+r)b_1} + \kappa\alpha\mu_1^P\right]$$

Similarly, the decentralized equilibrium allocations with taxes on debt are characterized by $\{c_t, h_t, b_1, b_2, p_t, \mu_1, \tau_1, \tau_2\}_{t=0,1,2}$ given initial debt $b_0 = 0$ and $\tau_0 = 0$ such that the representative household optimizes, $\mu_1 \geq 0$,
$T_t = \tau_t(1+r)b_t,\ t = 1, 2$, and the budget and resource constraints are satisfied.

Setting the taxes to

$$\tau_1 = \frac{E[\mu_1^P]\kappa\alpha}{E[\frac{1}{c_1}]}$$

$$\tau_2 = \frac{(\mu_1^P(1-\kappa\alpha) - \mu_1)c_2}{\beta(1+r)}$$

where all variables are evaluated at the constrained efficient allocations and borrowing choices. Redistributing the tax revenue as a lump sum transfer equates the conditions characterizing the decentralized and constrained efficient equlibria, implementing the planner's allocations into the decentralized economy.

(ii) Extending the argument to the recursive, transformed model, the planner's allocations are characterized by

$\{c_t, h_t, b_{t+1}, p_t, \mu_t^P\}_{t=0,1,2,\ldots}$, given initial debt $b_0 = 0$ such that the planner's euler equation is satisfied, $\mu_t^P \geq 0$, and the resource constraints and market clearing price hold for each t.

The planner's euler equation in the full model is given by

$$\frac{1-\alpha}{\tilde{c}_t} = \beta(1+r)E\left[\frac{1}{e^{\epsilon_{t+1}}}\left(\frac{1}{\tilde{c}_{t+1}} + \kappa_h \frac{\alpha}{1-\alpha}\mu_{t+1}^P\right)\right] + \left(1 - \kappa_h \frac{\alpha}{1-\alpha}\right)\mu_t^P$$

The decentralized equilibrium allocations are characterized by
$\{c_t, h_t, b_{t+1}, p_t, \mu_t, \tau_{t+1},\}_{t=0,1,2,\ldots}$ given $b_0 = 0$ and $\tau_0 = 0$, such that the representative household optimizes, $\mu_1 \geq 0$,
$T_t = \tau_t(1+r)b_t,\ t = 1, 2$, and the budget and resource constraints are satisfied.

where the household's euler equation is

$$\frac{1-\alpha}{\tilde{c}_t} = \beta(1+r)E\left[\frac{1}{e^{\epsilon_{t+1}}}\frac{1}{\tilde{c}_{t+1}}\right] + \mu_t$$

Evaluating all variables at the planner's allocations and borrowing choices leads us to the following expression for taxes in period t:

$$\tau_t = \frac{\frac{1}{\beta(1+r)}\left(\mu_t^P\left(1 - \kappa_h\frac{\alpha}{1-\alpha}\right) - \mu_t\right) + E_t\left(\frac{\mu_{t+1}^P \kappa_h \frac{\alpha}{1-\alpha}}{e^{\epsilon_{t+1}}}\right)}{E_t\left(\frac{1}{e^{\epsilon_{t+1}}\tilde{c}_{t+1}}\right)}$$

The corresponding lump sum transfer in period $t+1$ is $T_t = \tau_t(1+r)b_t$, or equivalently in transformed notation, $\tilde{T}_t = \frac{\tau_t(1+r)}{e^{\epsilon_t}}\tilde{b}_t$.

■